Modelling the Processes of Maximizing Hotel Revenues, Based on Applying the Linear Programming and the Network Flows

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Abstract. This work proposes to solve a problem related to maximizing hotels' revenues through two methods established in operational research domain. In the first part of the paper, the approach involves formulating the objective function and problem's constraints, as well as the expansion of the model, taking into consideration clients' preferences and the opportunities of group reservations. In the second part of the paper, the problem is solved with the help of network flows model, which allows optimum allocation of the rooms in real time. At the end of the paper, there are highlighted the advantages of applying those two mathematic methods within the strategies of performances development within hotel industry.

Keywords: hotel industry, optimisation, linear programming, network flows

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1. General considerations regarding applying linear programming and network flows in economics domain

The managers of the multinational hotel chains must adopt a balanced attitude in the management of the cultural differences. Many times, it is necessary to reconcile the differences to a certain degree, by acknowledging them and adapting the employees to others' habits. An exaggeration attitude of the cultural differences can be in the disadvantage of communication and collaboration between employees in a global dynamic and sophisticated business environment.

In order to model the processes of maximizing hotel revenues, we will approach two methods frequently used in operational researches, namely linear programming and the theory of network flows.

Linear programming represents an efficient method that includes more decision variables within one single model, in order to establish the optimum solution. The models of linear programming have a very important place both in economic theory and practice. The operational researches benefit from the input of the interdisciplinary approach that allowed the thoroughness of the complex systems' maximal efficiency analysis and the discovery of new concepts of the strategy of resources optimum allocation, approach that improved the research and knowledge methods, and the managerial practice was enriched with an instrument very useful to economical analysis and grounding decisions.

The structure of a linear programming problem includes an objective function, decision variables, constraints and parameters. The objective function implies planning a set of performance parameters described through decision variables. The constraints are represented by linear equations that describe the restrictions regarding the resources available, while the parameters are designated by the numerical values that describe the allocation of resources.

The networks represent graphs in which to each arc it is associated the arc's capacity and in which there is only one input point and only one output point. The method of network flows presents the following characteristics:

- There are given many sources that have limited resources and many destinations that request certain quantities from the resources from different sources;
- The transfer of resources from sources to destinations is performed by going through certain intermediary centres that connect resources to destinations, and between each two centres there is a certain limit of quantity that can be transported (capacity restrictions);
- The objective tracked is the transfer of a maximum volume of resources;

- After performing the maximum transfer of resources, it is analysed thenceforth if it is possible to be satisfied all requests and how the network graph should be organized;

We will apply linear programming in a problem specific to performance management in hotel industry and we will highlight that the optimum solution can be approached also by formulating a network flows problem.

Designing the mathematic model of maximizing hotel revenues and implicitly, of increasing its performances, will start from the hypothesis that there is a transparency of information regarding clients' requests, classified according to the date of arrival, duration of stay, tariff according to the type of room.

2. Designing and testing the model regarding maximizing hotel revenues based on applying the linear programming and network flows

Formulating the problem consists in finding the optimum mix of allocating the rooms of a hotel so that to maximize the revenues from its sales. We will not take into consideration the additional revenues coming from the restoration services, renting the conference spaces, etc.

The mathematic model can be described through the following variables:

i, l, j – time indices („i" and „l" refer to the arrival date of a client, while „j" represents the date of his/her departure from the hotel; „l" represents the arrival date before the date provided through the booking „i", due to a certain context);

p – tariff indices according to the type of room;

$x_{i,j,p}$ - the number of rooms sold at the tariff „p" to the clients of a hotel that arrived on „i" day and left on „j" day (i<j);

c_p - the tariff according to the type of room;

K_i - the maximum capacity of a hotel in "i" day;

- the request for the rooms sold at „p" tariff to hotel's clients that have arrived on „i" day and left on „j" day;

λ – the maximum duration of clients' stay within the hotel.

Choosing the value implies formulating the following objective function:

$$\text{Maximization} \sum_{i,j,p} (j-i) c_p x_{i,j,p} \qquad (1)$$

Which will be subject to the constraints:

$$\forall i: \quad \sum_{l \le i} \sum_{i<j} \sum_{p} x_{i,j,p} \le K_i \qquad (2)$$

$$0 \le x_{i,j,p} \le d_{i,j,p} \qquad (3)$$

$x_{i,j,p}$ - round number (4)

If a client of a hotel books a room at tariff from „i" day up to „j" day, then his/her contribution to hotel's revenues will be expressed through the variable described in the equation (1): (j – i).

Every day, the hotel's total number of clients cannot exceed its maximum capacity, but only in the case when the informatics system of recording the bookings generates errors and situations of over allocation appear. In this case, clients have to have ensured rooms at other hotel hereby, having the same class, the costs being bore by the hotel that made bookings over its maximum capacity of rooms available.

The third constraint refers to the fact that a hotel cannot sell more rooms than it is the value of the request expressed by the clients, while the forth constraint reflects the fact that there cannot be sold fractions from the rooms available from a hotel.

We will extend this mathematic model, taking into consideration clients' preferences according to the types of rooms from a hotel (single, double, suite). We suppose that there are preferences that reflect the hotel's differentiated approach of the tariffs. For example, a client might prefer a room of type 1 over a room of type 2, even if the tariff for the room of type 1 is higher than the one for room 2. A new variable that can be included in the model is represented by the maximum capacity of rooms available at tariff „p" - . In case a hotel has a low request for rooms of type 1 and a higher request for rooms of type 2, than it will

concentrate the marketing efforts in order to attract clients on the segment with a lower request so that to be able to maximize its revenues and implicitly, the profit.

Introducing this new variable within the model will generate the following constraint:

$$\forall i, \forall p: \quad \sum_{l \le i} \sum_{i < j} \sum_{m \le p} x_{i,j,p} \le \sum_{m \le p} K_i \qquad (5)$$

Also, we suppose that besides the tariffs differentiated for the different types of rooms, a hotel has different tariffs for rooms of the same type (for example, it offers smaller tariffs to preferential clients or to the ones which are recorded in a fidelity program). This way, we will introduce the variable r(p) that designates the tariffs differentiated for a room available at tariff „p" for all categories of clients. In this situation, the inequation (5) becomes:

$$\forall i, \forall p: \quad \sum_{l \le i} \sum_{i < j} \sum_{m \le p} \sum_{c \le r(p)} x_{i,j,p,c} \le \sum_{m \le p} K_p \qquad (6)$$

where highlights the number of rooms of „m" type sold at „c" tariff to the clients of a hotel that arrived on „i" day and they stay until „j" day.

Another extension of the mathematic model takes into consideration the group booking, which implies the availability of the information regarding the following variables:

i* - Day of arrival of the group of tourists at the hotel;

λ_g - Duration of stay of the group of tourists;

μ_g - Size of the group of tourists;

c_g - The tariff practiced by the hotel for groups;

x_g - Dichotomist variable that indicates group's acceptance or rejection by the hotel (take "0" value in case of rejection and "1" in case of acceptance).

The objective function of maximizing hotel's revenues becomes in this situation:

$$\text{Maximization} \sum_{i,j,p} (j-i) c_p x_{i,j,p} + \lambda_g \; c_g \; \mu_g \; x_g \qquad (7)$$

This function of optimization is subject to the following constraints:

$$\forall i \in \{i^*, \ldots\ldots, i^* + \lambda_g\} : \sum_{l \le i} \sum_{i < j} \sum_{p} x_{i,j,p} + \mu_g x_g \le K_p \qquad (8)$$

In practice, the tariff used by hotels for tourists' groups is not usually predetermined, but mostly negotiated. During negotiations, hotel manager must have in view the minimum price that he/she can accept in case of selling the rooms to a group.

Applying the linear programming in order to maximize the revenues of a hotel has certain limits that relate to the large number of decision variables. For example, if a hotel allows bookings with at least 6 months in advance, with a maximum duration of 2 weeks, for the 5 types of different tariffs, the model based on applying the linear programming will involve over 60 variables.

Formulating the same problem of maximizing the revenues of a hotel with the help of network flows method, will allow optimum allocation of the rooms in real time and will considerably reduce the number of variables involved in linear programming.

Given G = (V,A), a graph that is characterized through multiple arcs. V represents the source node, while A represents the destination node. The arcs of type connect "i" and "j" nodes, if there are complied with the conditions: $j - i \le s$ and $i < j$. The number of multiple arcs between any two nodes is determined by the number of different tariffs practiced by the hotel, according to the type of room.

We will define the function of profit ϕ on the network graph, as follows:

$$\forall (i,j)_p \in A: \quad \phi[(i,j)_p] = \begin{cases} (j-i)c_p : j-i \le s \\ 0 : i > j \end{cases}$$

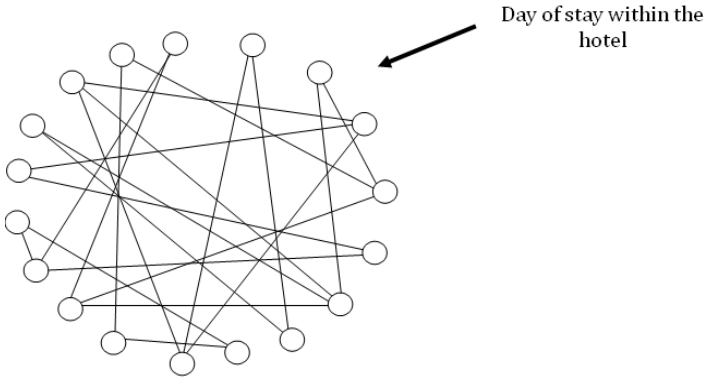

Figure no. 6.1: Example of graph G = (V,A)

The nodes within the network represent the days of stay of hotel's clients. The flows associated to the arcs disclose the number of rooms sold at the tariff „p" to the clients that arrive in „i" day and leave the hotel in „j" day.

The flows induced through arcs oriented from the left to the right side of the variable (the number of rooms sold at the tariff „p" to the clients of a hotel that arrived in „i" day and left in „j" day) are conditioned by the variable (the request for the rooms sold at tariff „p" to the clients of the hotel that arrived in „i" day and heft in „j" day). The flows induced through arcs oriented from the right to the left side of the variable (the number of rooms sold to clients that arrive at the hotel in two consecutive days) are conditioned by the variable (the maximum capacity of a hotel in "i" day).

The objective that we propose consists in finding the flow within the network that maximizes hotel's revenues. Conserving the flows in each node of the network implies constraints similar to the ones reflected within applying linear programming through the inequation (2): $\sum_{l \le i} \sum_{i<j} \sum_{p} x_{i,j,p} \le K_i$

Except for the source and destination, the flow quantity that enters within a node is equal to the flow quantity that exits from a node. Because we can generate the constraints in the nodes „i – 1" in inequation (2) through the first „i – 1" constraints of conserving the flows, we can deduce that the „i – 1" constraint resulting from conserving the flows in the node „i" will generate the constraint „i". (Figure no. 2).

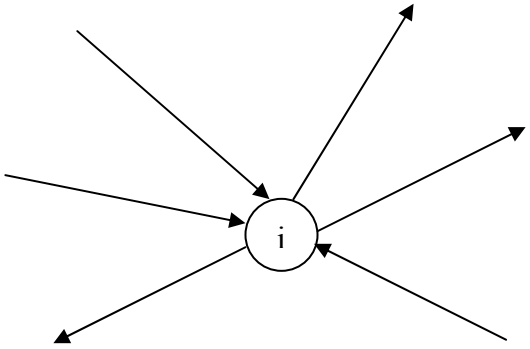

Figure no. 6.2: Model of conserving the flows in node „i"

Including the „i – 1" constraint in the inequation (2) becomes:

$$\sum_{l \le i-1} \sum_{i-1<j} \sum_{p} x_{i,j,p} = x_{i,i-1} \le K_{i-1} \qquad (9)$$

Conserving the flows in node "i" will generate the equation:

$$\sum_{l<i} \sum_{p} x_{l,i,p} + x_{i+1,i} - x_{i,i-1} - \sum_{i<j} \sum_{p} x_{i,j,p} = 0 \qquad (10)$$

Introducing the variable from the equation (9) in the equation (10) and taking into consideration the principle of conserving the flows, it provides the same result which corresponds to the constraint (2) from formulating the problem with the help of linear programming method.

3. Conclusions

The services in the hospitality industry are characterized by a high level of direct interactions between persons of different nationalities. If the staff of a hotel, as a hospitality service provider and most of his customers come from different cultural contexts, then the performances will be conditioned by the way in which the customers perceive the services both from a financial perspective (quality – price report) and especially a cultural one (they appreciate if their traditions, habits and values are respected). The managers of the multinational hotels must evaluate the cultural needs of the different categories of customers, coming from various countries in order to carry out a cultural synergy and the employees, especially those from the Front–Office must prove cultural intelligence abilities in the interactions they have with such categories of customers.

The performances of the multinational hotel chains are closely connected to the managers' abilities to understand the cultural differences specific to the customers, employees, providers form every external location, to use analyses of the cultural profiles in the policy of public relations and to promote the cultural synergy.

The multinational hotel chains consider that the employment of some intelligent persons from the cultural perspective represents a major competitive advantage, considering the present businesses and adopt cultural diversity as part of the human resources strategy.

These two mathematic methods applied for increasing the performances of a hotel, based on maximizing the revenues (linear programming and network flows method) lead to the same results, thus validating the model designed. The main advantage of this mathematic model consists in its capacity of personalizing to the specific of the activities of each hotel, generating a mix of optimum allocation of the rooms in order to maximize the revenues and implicitly, to increase the performances.

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